Mathematics – from Potential to Strength

MATH POST

Type of funding scheme: Coordination and support actions (Support)

Work programme topics addressed: FP7-REGPOT-2010-1

Coordinating person: Prof. Dr. Nataša Krejić

Participant: Center for Mathematical Research of Nonlinear Phenomena, Department of Mathematics and Informatics, Faculty of Sciences University of Novi Sad

Table of Contents:

1. Scientific and/or technical quality, relevant to the topics addressed by the call

1.1. Concepts and objectives

Centre for Mathematical Research of Nonlinear Phenomena, CMRNP is a research unit at the Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad (www. http://www.dmi.rs/Projects/CMRNP/). The Faculty (UNSPMF) consists of five departments covering all natural sciences, mathematics and computer science and presents interdisciplinary environment dedicated to high quality research and teaching. Faculty of Sciences represents the regional center for natural sciences and mathematics, and is the only one of a kind in the region of Vojvodina (north province of the Republic of Serbia). Capital of Vojvodina, Novi Sad, is the University center and gathers large number of educated people and experts. Faculty has 500 employees, 4000 undergraduate and 400 postgraduate students. Numerous international projects are being conducted at UNSPMF at this moment. UNSPMF has coordinated 3 FP6 projects (REP LECOTOX, CECRA, RPR –CMEP), acted as a partner in 2 FP6 projects (ADAGIO, ALARM) and thus posses necessary administrative capacity for supporting this project. UNSPMF is a partner at one FP7 project at this moment.

The Department of Mathematics and Informatics - **DMI**, has 48 permanent teaching and research staff, offers bachelor, master and PhD degrees in mathematics and computer science, coordinates 6 research projects financed by the Ministry of Science and Technological Development of the Republic of of Serbia (MNTR or the Ministry from now on), participates in several international research and networking projects and has established scientific and educational cooperation with relevant mathematical departments and departments for computer science departments in Europe, SAD, Asia and South America. Three Tempus projects were coordinated and implemented by DMI, one of them resulting with the first Serbian master programme in Applied Mathematics ready to answer the needs of society and in particular economy in transition process. The programme has been developed in cooperation with European Consortium for Mathematics in Industry (ECMI – http://www.ecmi-indmath.org). A number of initial steps towards cooperation with other stake holders are already implemented. Besides the traditional courses offered to teachers, we also offer industrial training courses in cooperation with the National Bank of Serbia and SECCF. Further development of collaboration with industry and SMEs which leads towards new research tasks and projects is one of the future goals of DMI as a whole.

CMRNP gathers researchers from 3 research groups at DMI financed by MNTR. In May 2008 the Center was recognized by the Ministry of Science and Technological Development and the National Council of Science as a **national center of excellence**. The principal aims of CMRNP are to establish closer cooperation among different research teams, increase the level of scientific results, promote the cooperation between mathematics and applied sciences and technology, stimulate interest for science among young people and further develop international cooperation. Joint human potential with overlapping research interest and cooperation of researchers with different skills is expected to lead to new significant results. The Center has 21 permanent PhD researchers and 23 PhD students.

The Center is the leading mathematical research unit in Serbia with **strong regional** visibility. Concentration of high quality researches in several mathematical areas together with dedicated and enthusiastic researchers creates suitable environment for excellence and stimulates creativity necessary for cutting-edge research. We are actively participating at several regional projects and DAAD programme of Centers of excellence in applied mathematics in South East

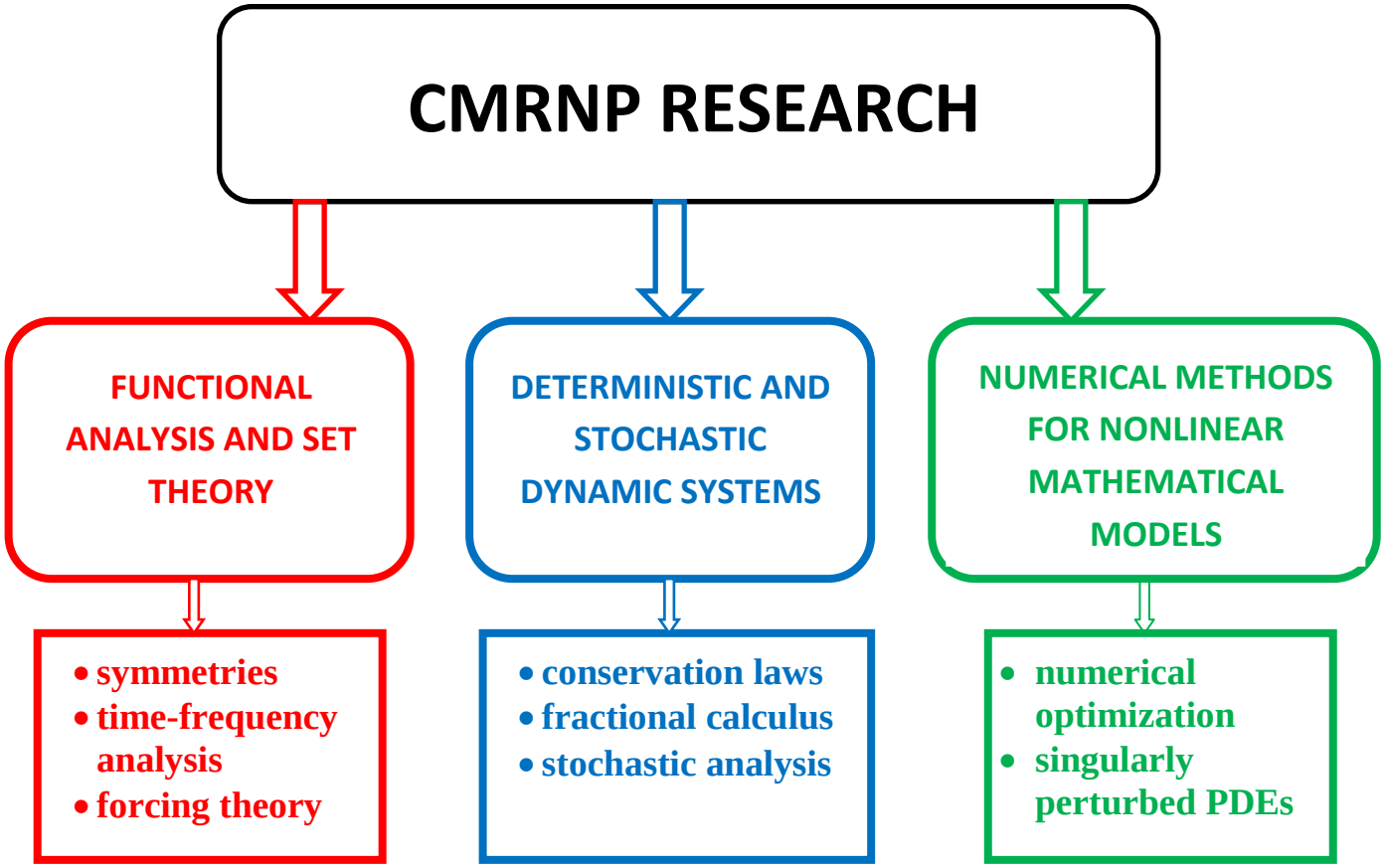
Europe. In the last couple of years we have organized 5 intensive courses for young researchers from WB countries.

The existing cooperation includes researchers from many European, Asian and South American universities. A number of international events have been organized and several international projects have been successfully coordinated by the members of the Center. Further development of international cooperation is an important goal of the Center. Unlocking the potential of Center with effort proposed in this project would be beneficial to ERA given that existing contacts and cooperation will be permanently transformed into sustainable network and thus provide synergy of research capacities for further scientific development. The Center is also making pioneering efforts in Serbia of applying advanced **mathematical knowledge and research in industry**.

Serbia is in the process of determining **strategic policy in science**. The document (http://www.nauka.gov.rs/cir/images/stories/vesti/09-06-29/strategy_presentation_3.pdf) lists several research priorities that are relevant to the research performed at the Center, in particular energy efficiency, biotechnology and environmental research. Nonlinear phenomena are present in these research themes and advanced mathematical research will certainly yield important advancement in the respective areas. Hence further development of the Center into highly visible point on ERA map would be of importance for the country as a whole.

The research conducted at the Center covers a broad range of topics from both theoretical and applied mathematics. The researchers are concentrated into 3 broad research groups that are mutually connected and thus provide opportunity to synergy and creation of added value in cooperation. The principal investigators at the Center are leading research teams consisted of 21 permanent researchers and relatively large number of young researchers (mainly PhD students). The Center has two weekly seminars – “Seminar for Functional Analysis, PDEs and Topology” and “Seminar for Numerical Mathematics” with numerous of foreign visitors. The purpose of these seminars is to serve as a forum for presentation of new ideas and results, provide discussion and facilitate the cooperation and exchange of knowledge among different research teams. The head of The Center is Prof. Dr. Stevan Pilipović, and project coordinator Prof. Dr. Nataša Krejić is the coordinator of scientific activities in the Center.

The main research topics and research teams with short profiles of the key researchers are described below.



Functional analysis and set theory – FAST GROUP

Team members: Dr. Stevan Pilipović, Dr. Sanja Konjik, Dr. Nenad Teofanov, Dr. Miloš Kurilić, Dr. Milan Grulović, Dr. Aleksandar Pavlović, Dr. Boris Šobot, 5 PhD students.

Dr Stevan Pilipović, member of the Serbian Academy of Sciences and Arts. Born 1950. PhD at UNS, 1979. He has published 180 papers in international peer-reviewed journals, 6 monographs and 3 volumes of Lecture notes. He was visiting professor at University of Tokyo, University Paris VII (11 times), University of Vienna. He serves as a member of the National Council for Science since 2006, and as an editor in several Serbian scientific journals as well as in two international journals: Pseudo-differential operators and Integral Transform and Special Functions. So far he has supervised 22 PhD. He has been coordinating several international projects (USA, Austria, France, Germany, UNESCO) and now coordinates several domestic research projects. His main research results include the development of local and microlocal analysis within ultradistribution spaces, pseudodifferential calculus in algebras of generalized functions, abstract stochastic calculus through chaos expansion, equations with singularities, time frequency analysis through Gabor frames and wavelets.
Dr Sanja Konjik. Born 1976. PhD thesis in Symmetry group analysis in non-smooth settings, defended at University of Vienna, 2008. Main research interests include symmetry group analysis, algebras of generalized functions, calculus of variations, fractional calculus. She was visiting researcher at University of Vienna several times and has published 9 research papers.
Dr. Nenad Teofanov. Born 1968. PhD thesis in pseudodifferential operators on ultra-modulation spaces, 2000. Main research interests are in harmonic analysis and time-scale

analysis. Visiting researcher at the University of Vienna and University of Torino; junior research fellow at the Erwin Schrödinger Institute, Vienna. Author of 18 papers.
Dr Miloš Kurilić. Born in 1960. PhD thesis in topology, 1994. Main research interests in set theory, topological model theory and general topology. At present coordinates one domestic research project and one international. A member of editorial board of Publication d'el Institute Mathematique (Belgrade). Has published 30 papers and supervised 2 PhD theses.

The main directions of this research group are originating from strong orientations towards functional analysis, topology and set theory. The following three topics are the main interest of this group: symmetries, time-frequency analysis and models of set theory. Details of each subgroup are given below.

Mission and objectives of the FAST Research group.

- Research towards synthesis of two mathematical fields of significant current interest: symmetry group analysis of differential equations and geometric theory of generalized functions.
- Developing time-frequency and time-scale analysis as parts of harmonic analysis which flourishes in the last two decades, in particular application to the reconstruction of climate and environmental dynamics and in wireless communication systems.
- Models of set-theory contain all classical mathematical objects as numbers, complex functions and differentiable manifolds. The stability of old and the appearance of new objects in models of set-theory obtained by forcing are investigated, as well as the infinite games on Boolean algebras and topological structures induced by order. Model-theoretic forcing, reduced products and ultraproducts are explored as well.

Research programme in relation to the state-of-the-art in the research field.

-The methods of Lie group analysis ([Ol]) provide powerful tools for examining PDEs. Recently, such methods have been extended to encompass non-smooth solutions using algebras of generalized functions ([KO]). They provide good framework for applications in general relativity, nonsmooth mechanics, nonlinear distributional geometry or global analysis (see [RT]).
- The results in functional analysis, time-frequency and time-scale analysis, theory of generalized functions, pseudo-differential operators, Fourier integral operators and microlocal analysis, [Do], [Fe], [Gr] and [Me]. Recent developments in the field, closely related to the research programme, are asymptotic analysis [Vi], microlocal analysis [Mo], wireless communication systems [St].
- Results of Jech, [G1] on "cut-and-choose" games on Boolean algebras motivate exploring and comparing old and new games and finding the algebraic, forcing and measure-theoretic interpretation of these results. The Control Measure Problem imply the need to explore some topological structures similar to the sequential topology.

Major research achievements.

- The methods of Lie group analysis of differential equations were extended in [DKP] to weak solutions of linear and nonlinear PDEs while in [KK0] generalized group actions were studied on differentiable manifolds in the Colombeau framework, extending previous work on flows of generalized vector fields and symmetry group analysis of generalized solutions.
- Wavelet transform and time-frequency methods for the characterization of wave-front sets, [PTT]. Asymptotic analysis [PT]. Decomposition and continuity properties of pseudodifferential operators, [Te1]. Frame theory in Frechet spaces, [PST].
- The stability of several structures was investigated in [FS1]. Various types of games, [G1]. Boolean algebras, [STT2], [STT3]. Combination of forcing and reduced products, [MTF2].

Description of the current research and future research plans

- Research focuses on extending notions and main results from symplectic geometry, originating from the Hamiltonian formulation of classical mechanics, to the framework of Colombeau generalized functions as well as to the study of sheaf properties of generalized functions on manifolds. Research on connection between pseudo-differential operators and

symmetries [RT] will be done. Other goal is establishing links between symmetry group analysis, generalized Colombeau functions and fractional calculus.
- Characterization of wavelet transform and its inverse in spaces of ultra-distributions and the continuity properties of pseudo-differential operators whose symbols may have (almost) exponential growth and their micro-local properties. New constructions for frames in Frechet spaces. Representation theorems which combine the local and global behaviour of a signal. Investigation and understanding of wavelet transform, time-series arising in quaternary climate and environmental dynamics, pseudodifferential operators and frames with application to the equalization problem in wireless communication systems
- The research is focused on the effect of forcing on inseparable sequences. Infinite games on Boolean algebras as well as convergence structures on Boolean algebras are considered, as well as the relations between the corresponding topologies including the sequential topology.

Previous and existing research collaborations, both national and international. University of Vienna, University of Innsbruck, Universite Paris VI, Universita di Torino, Vaxjo University, Imperial College London, National Academy of Sciences of Ukraine, SANU, Department of Mechanics and Department of Geography at U Novi Sad.

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Deterministic and stochastic dynamical systems - DSDS Research Group

Team members: Dr. Marko Nedeljkov, Dr. Stevan Pilipović[1], Dr. Danijela Rajter-Ćirić, Dr. Dora Seleši, Dr. Mirjana Stojanović, Dr. Arpad Takači, Dr. Đurđica Takači, 9 PhD students

Dr. Marko Nedeljkov. Born 1966. PhD thesis in Generalized functions and PDEs, 1995. Main research areas: systems of conservation laws, mathematical fluid dynamics, generalized and numerical solutions to PDEs and stochastic PDEs. He has published 34 papers. Dr. Nedeljkov has given invited lectures at Institut für Technische Mathematik, Geometrie und Bauinformatik, Innsbruck, Edwin Schrödinger Institut Wien, Austria, and Serbian Academy of Sciences and Arts. So far he has supervised 2 MSc theses and 1 PhD thesis and is supervising 2 PhD students now. At present he serves as Director of the Department for Mathematics and Informatics, UNSPMF.
Dr. Danijela Rajter-Ćirić. Born 1973. PhD thesis in deterministic and stochastic differential equations in Colombeau generalized function spaces, 2002. Main research interests include stochastic analysis, theory of generalized functions, stochastic partial differential equations. She was visiting researcher at University of Innsbruck (Austria). Has published 12 papers.
Dr. Dora Seleši. Born 1978. PhD thesis in stochastic PDEs in infinite dimensions, 2007. Main research interests include white noise analysis with applications to generalized stochastic processes and SPDEs. She was visiting researcher at University of Innsbruck 2005, and visiting lecturer at Brown University 2009 supported by ARO MURI Grant "Spatial-Temporal Nonlinear Filtering". Has published 9 papers, supervised 6 MSc theses and currently is supervising 1 PhD student.
Dr. Arpad Takači, born 1951. Main research interests are linear and nonlinear PDEs, modeling and simulation, wavelet theory, operator and fractional calculus. As a Fulbright scholar, he spent a school year 1989/90 at the VCU. He has coordinated an EU funded project entitled “Development of Computer-aided Methods in Teaching Mathematics and Science” and WUS grant the project entitled “Mathematical Modeling” in 2005.

Mission and objectives of the DSDS Research group.
There are three main directions within this group: conservation laws, fractional calculus and stochastic analysis. The main objectives are:
- To analyse and solve problems in conservation law PDEs theory arising in fluid dynamics and chromatography. To improve a general knowledge about non-classical solutions or to numerically compute them for 1-D problems. To investigate classes of multidimensional problems, especially the ones with large applicable value.
- Stochastic differential equation involving singular coefficients, singular data and singular initial values represent many physical, economical, biological and even social phenomena. To solve these equations one needs the concept of a generalized random process. The research objectives include solving SPDEs driven by singular stochastic processes, the analysis of the structure of generalized stochastic processes with emphasis on chaos expansions, and applications to financial and actuarial mathematics, mathematical physics, mechanics.
- Many real word systems can be modelled by replacing common derivatives and integrals by non-integer order ones, which make the core of the fractional calculus. Thus, the objective is to introduce, analyse and solve certain mathematical models, in particular viscoelastic problems.

Research programme in relation to the state-of-the-art in the research field.
- In recent years the interest in so called non-classical solutions of conservation laws is arising, [CL], [KK]. Multidimensional problems in gas dynamics are still waiting to be solved, [B-GS].

[1] CV provided within FAST group

Some numerical schemes are proved to provide discrete shock profiles for certain equations or systems and some of them are found to be appropriate for capturing more singular objects than shock waves, [Se]. Explicit ("real-world") applications with non-classical solutions are intensively studied, [Maz]. Our research will be focused on such problems.
- White noise theory, as a discipline of infinite dimensional analysis, [HKPS] had a fast development due to its broad spectrum of applications. In [OR], non-linear stochastic differential equations are solved by regularization methods in generalized function algebras. In [HOUZ], equations are solved using the Wick product and the Hermite transformation, while in [LR] the propagator method is used to transfer the SDPE into an infinite system of PDEs. Our research focuses on singular SPDEs, linear and non-linear, related to all these fields and methods.
-There are two approaches in formulating DE with fractional derivatives as models in various fields (viscoelasticity, fluid mechanics, diffusion, waves): variational principles and fractional derivative models with memory effects, [Po], [SKM].

Major research achievements.
- The results with different solutions concepts for delta and singular shocks of 1-D systems of conservation laws, [Ne0]. The interaction problems are contained in [Ne1], [NO]. A numerical verification ([KKN]) of delta shocks for pressure-less gas dynamics system. An entropy criteria and a solution to a general delta shock interaction problem in [Ne2].
- Solutions of wide classes of SPDEs driven by white noise and other singular processes are obtained in the framework of generalized function algebras in [NR]. In [PS1], we obtained a generalization of the Wiener-Ito chaos expansion for generalized stochastic processes. The work involving Gaussian processes is continued in [MPS]. In [PS2] we proved uniqueness of the solution to a very general class of singular second order SPDEs driven by an elliptic operator.
- In the papers [St], and [TT] we solved different classes of fractional differential equations. In [APZ], we investigates conditions for solutions of fractional variational problems.

Description of the current research and future research plans
-Solutions and their properties for conservation laws containing delta shocks, their admissibility and interactions with other waves are started to be analysed (analytically and numerically). Numerical and analytical shock structure properties (discrete shock profiles) research is started. Preliminary results about non-classical solutions to some multidimensional problems are obtained. The future research plan is to continue these investigations.
-We are focusing on solving stochastic partial differential equations in the framework of generalized stochastic processes. All equations considered are highly applicable to the problems where modelling of spatial-temporal processes is required. Future work will be on the unified theory of chaos expansion and Colombeau methods for singular stochastic processes, implementing fractional derivatives and Malliavin calculus and some classes of SPDEs involving it. Extending the notion of the Skorohod integral, and solving related SPDEs.
-Research is directed towards the use of fundamental theory of functional analysis especially generalized functions theory, Schauder type theorems and Volterra and Fredholm theory for solving linear and nonlinear fractional equations. Moreover, the use of the local symmetries in the analysis of Lagrangian with fractional derivatives will be developed.

Previous and existing research collaborations, both national and international. University Innsbruck, University of Vienna, Bulgarian Academy of Sciences, Universite de Guayane (Guadeloupe, France), University of Berlin, University of Bologna, Cagliari and Torino, Brown University, Serbian Academy of Sciences and Arts, Department of Mechnics U Novi Sad.

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Numerical Methods for Nonlinear Mathematical Models - NM Research group

Team members: Dr. Nataša Krejić, Dr. Zorana Lužanin, Dr. Helena Zarin, Dr. Dragoslav Herceg, Dr. Đorđe Herceg, Dr. Sanja Rapajić, 9 PhD students

Dr. Nataša Krejić. Born 1966. PhD thesis in Numerical Analysis – linear and nonlinear systems of equations, 1994. Main research interests include nonlinear optimization, smooth and semismooth problems as well as applications of optimization techniques in industrial problems, in particular high frequency trading. Has published 48 papers. Dr. Krejić was visiting professor at the State University of Campinas, SP, Brazil, several times under grant from Fapesp, SP, Brazil, Senior Research Fellow at Central European University in Budapest and Erasmus Mundus visitor at CASA, TU Eindhoven. At present she is the coordinator of scientific activities at CMRNP, representative of UNSPMF at ECMI Educational Committee, head of Master programme in Applied Mathematics at UNSPMF and the leader of research project "Numerical Methods for Nonlinear Mathematical Models" – financed by the Ministry of Science and Technological Development of Serbia, 2006-2010. She was the coordinator of Tempus CD JEP project "Mathematics for Technological Development", 2003-2006 and research project for

Dresdner Kleinwort Securities, London “Modelling and Forecasting Stock Price Behaviour – Analysis of High Frequency Financial Data” 2006-2007. So far she has supervised 8 MSc theses and 1 PhD thesis and right now is supervising 5 PhD students. Besides that Prof. Krejić is engaged at at Quantitative Finance programme at University of Belgrade. From 2006 she serves as a member of the Index Committee at Belgrade Stock Exchange.

Dr. Zorana Lužanin. Born 1967. Main research interests are iterative methods for solving nonlinear systems of equations, iterative methods for problems of nonlinear (deterministic and stochastic) optimization, applications of numerical methods in economy and modeling. She was visiting researcher at OCIAM, University of Oxford. Author of 27 papers and supervised 3 MSc theses.

Dr. Helena Zarin. Born 1974. Main research interests are in numerical analysis of PDEs, in particular finite element methods for singularly perturbed problems. Author/coauthor of 20 research papers. Research assistant at Dresden University of Technology, Germany (06/2001-02/2003) under DAAD and DFG grants. Supervised 1 PhD and 1 MSc thesis.

Mission and objectives of NM Research group

This research group focuses on numerical methods for nonlinear mathematical models, in two directions in particular: numerical nonlinear optimization and numerical methods for singularly perturbed PDEs. Both considered groups of problems arise from mathematical models in a wide range of phenomena. The main research goal is to design methods with sound theoretical properties and good computational behavior in terms of convergence speed and computational effort.

Optimization problems that are of interest are: 1) semismooth problems arising in equilibrium models and 2) stochastic problems. The methodology used for solving semismooth problems is based on continuous optimization techniques, in particular on Jacobian smoothing approach and generalized Newton method. Problems with stochastic objective function and/or stochastic constraints are either models of phenomena in noisy environment or represent expectation of functions that depend on random parameters. The principal difficulty is the amount of computation due to the need of extensive simulations. Two main approaches – stochastic approximation (SA) method and sample average approximation (SAA) in combination with advanced knowledge of deterministic techniques are pursued in this area.

Singularly perturbed problems (SPPs) form a wide class of parameter-dependent ordinary and PDEs. We aim to derive various analytical properties including derivative estimates and solution decompositions. Main research goal is a construction and analysis of numerical methods for SPPs. An important effort is devoted to stable and robust procedures. We are in particular interested in application of layer-adapted meshes of various types, improvement of accuracy of approximate solutions implementing postprocessing procedures as well as various stabilizations.

Research programme in relation to the state-of-art in the research field.

The main research direction includes developing efficient methods for large-scale semismooth problems which are the models of important phenomena [FP] using different smoothing techniques in combination with classical optimization methods [F,KP,FS,C]. Our aim is to develop methods that will be able to deal with large scale semismooth problems with semismoothness present in objective function and/or constraints, in particular in problems of VaR like type that arise in financial and energy distribution models. Stochastic problems still represent a class where fundamental issues remain open, in particular for nonlinear models [RS]. The needs for extensive simulations, by Monte Carlo method or similar approaches, [LXF] as well as nonlinearity make them difficult both from theoretical and computational point of view. Recent results include methods with less extensive simulations for some classes of problems, [DF,BCT]. Extending these results to other classes of problems, in combination with SA or SAA approach and globalization techniques like line search or trust region for constrained problems would be an important achievement we aim to.

Mathematical models that involve a combination of convective and diffusive processes are among the most widespread in all fields where mathematical modeling is important, [M]. In general, the corresponding mathematical model consists of differential equations where

highest-order derivatives depend on a perturbation parameter. Standard numerical discretizations often fail to resolve exact solutions in layer regions thus invoking a construction of special methods, [RST,FHM,MRS]. We are interested in a construction of finite difference [FHM,MRS] and finite element methods [RST,B] on layer-adapted meshes. In particular, we aim to extend analysis of discontinuous finite element methods [ABC] to certain classes of SPPs and to apply higher order collocation methods as an alternative for the efficient and fast approximate solution of differential equations.

Major research achievements.

Convergence theory of Quasi-Newton (QN) methods for smooth systems, [KL] unconstrained and constrained optimization [FKM] is developed. All proposed methods are analysed theoretically and proved to be numerically competitive in the case of large dimensional problems. Convergence theory is developed for a number of methods of QN type with Jacobian smoothing for NCP problems [KLR, BK]. A globally convergent inexact Newton method with nonmonotone line search rule is proposed and analysed in [KR].

In 1-D case, result [HH] has been devoted to higher-order finite difference methods on layer-adapted meshes. Stabilization of the standard Galerkin method in the aspect of discontinuous finite elements has been applied to convection-diffusion problems with different layers, [FTZ,Z]. Recovery has been developed in [FTZ]. A superconvergence property of the method on the Shishkin mesh in two parameter problems is proved in [TZ].

Current research and future plans.

The first main objective of efforts in nonlinear programming is to derive methods able to deal with the stochastic objective function and/or constraints. Investigating conditions that would allow for variable sample size methods in SA and SAA approaches as well as extensive computational simulation will be the main topic. Other topics pursued are semismooth problem with semismoothness of VaR type and large dimension. Such problems are arising in risk modelling - in finance and energy distribution models.

Problems with two perturbation parameters are solved numerically in the context of finite element methods, with special emphasis on discontinuous methods with interior penalties and local projection stabilizations. Experimental verification of theoretical results represents an important part of our research. We plan to derive a general framework for the analysis of collocation methods of arbitrary order for SPPs, posteriori error estimation, implementation issues and parallelisation. We also plan to expand the analysis of two-parameter problems to the case of different relations between parameters and data functions.

Members of this group have developed **international cooperation** with the Optimization group at the State University of Campinas, SP, Brazil, Optimization Group at the State University of Sao Paolo, Humboldt University in Berlin, Dresden University of Technology (Germany), Kent State University (USA), Otto von Guericke University in Magdeburg (Germany) and University of Limerick (Ireland). On a national level, we have a colaboration with Faculty of Technical Sciences UNS.

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The **EMERGING TOPIC** in CMRNP research activities is application of **Mathematics in Industry.** Given that the majority of research themes are highly applicable, a number of efforts are being made in order to develop contacts with local and international industry, SMEs and public organizations. One research project was conducted for Dresden Kleinwort Securities, London "Modelling and Forecasting Stock Price Behaviour – Analysis of High Frequency Financial Data" 2006-2007 (team members: Dr. Nataša Krejić, Dr. Marko Nedeljkov, and Dr. Zorana Lužanin). The results of this project were confidential and were not published. Combining knowledge of NM and DSDS groups we successfully modelled and solved the problem of optimal pumping schedule in water distribution system in Zrenjanin, a small city in Serbia. These results are published in [KK]. Intensive research on optimal execution in high-frequency trading is the subject one PhD thesis (Miles Kumaresan, supervisor Prof. Dr. Nataša Krejić) that is at the final stages and the first part of that research is published in [2]. The present research is focused on the problem of optimal execution strategy in the presence of fragmented liquidity. The group for Stochastic Analysis and group for Numerical Optimization is cooperating with the National Bank of Serbia in the field of risk measures and actuarial mathematics and several MSc thesis are already defended in that area. Further efforts are to be made in this project, heavily using the know-how of EU partners. The risk models that are subject of research of several groups (stochastic analysis, PDEs, optimization) are particularly important within the present socio-economic conditions in the region, given that already

developed models for advanced financial, energy and carbon markets cannot be directly applied in the region due to significantly different circumstances.

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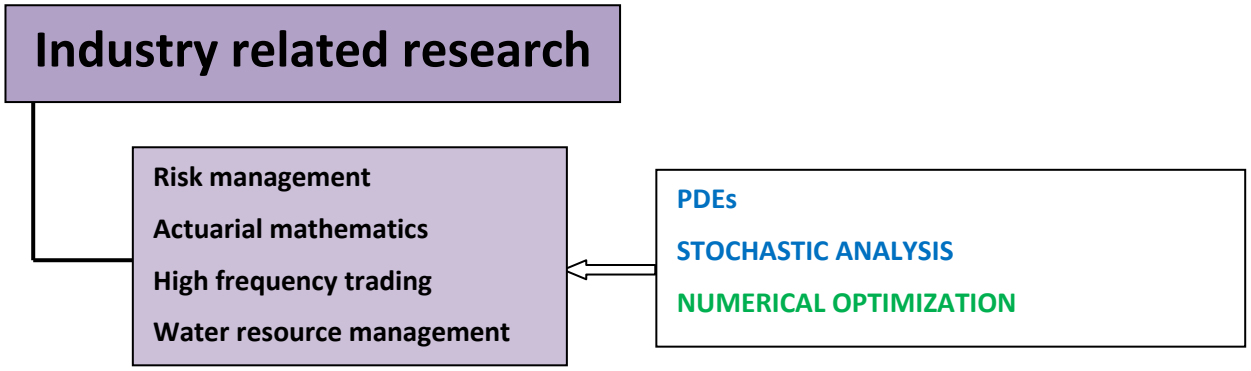
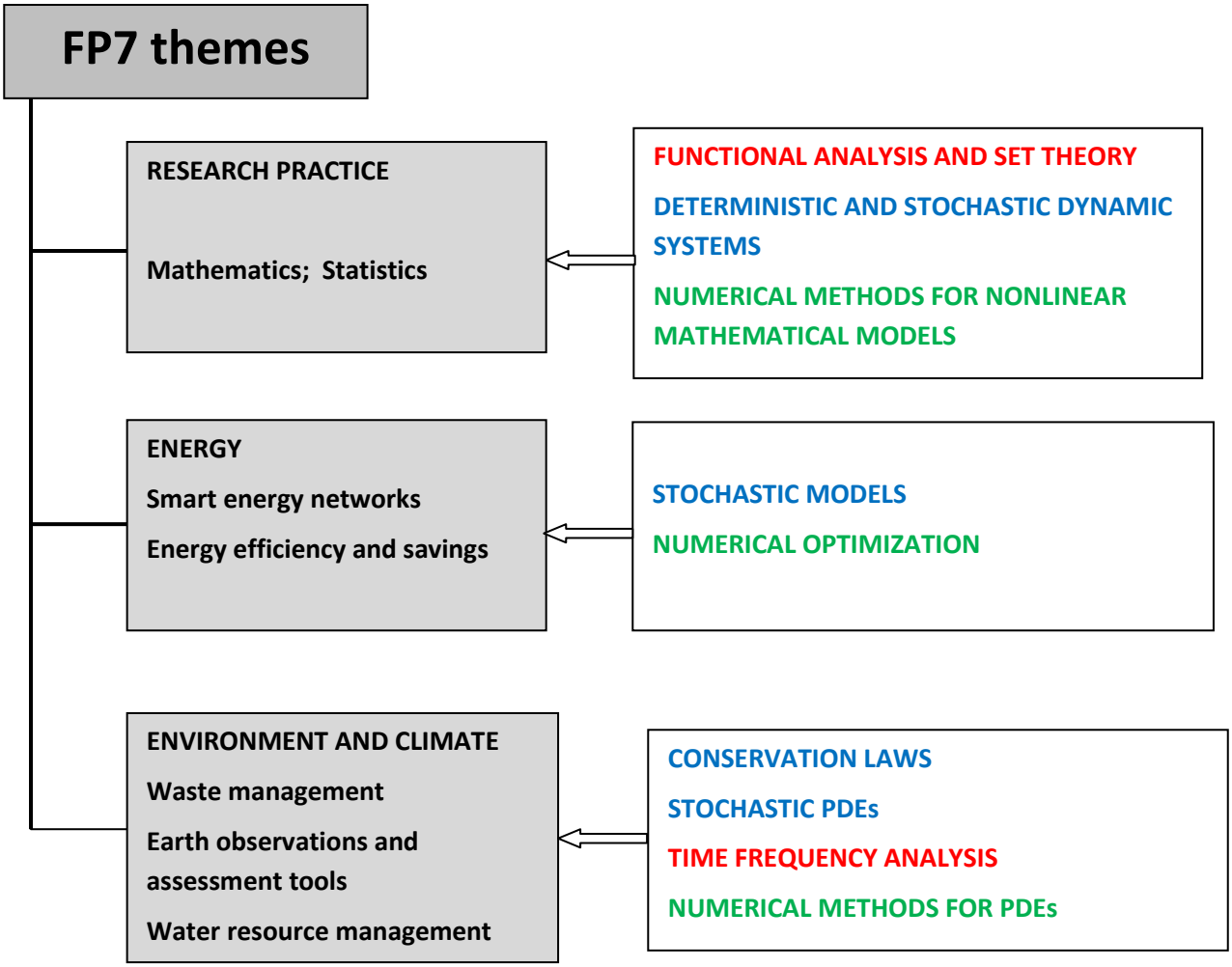


Current research potential of CMRNP. All research teams use the reasonably well equipped Library at the Department and have access to a large number of journals through e-system of the National Library provided by the Ministry of Science of Serbia. That is fairly good environment for theoretical research that is dominant in the FAST team. More applicable research conducted by the other two teams use standard computational equipment consisting of personal computers that do not allow massive computation, simulation and storage of data. A new computational laboratory able to provide parallel environment is thus necessary. The laboratory will be equipped with a server and 10 working stations forming a closed network. The software packages necessary for successful dealing with problems mentioned in future plans are Matlab and S+.

Relevance to FP7 themes. The principal theme for all research teams is Research in practice, Action 2 – **Mathematics; Statistics**. But the main strength of the Center in is concentration of people from different areas relevant to the applications – PDEs, optimization, stochastic, fractional calculus. Although the relevant models can be found in almost all FP7 themes, we will focus here on those coming from **Energy** and **Environment and Climate**. Mathematical models of energy distribution can be formulated using nonlinear optimization models with constraints. Depending on aims of a model, semismoothness or uncertainty, or even both phenomena, naturally appear. The energy distribution model has to be of large dimension to represent reality in the region or even more in European energy networks and hence large scale optimization techniques are needed. Relevant actions within the theme are Smart energy Networks and Energy Efficiency and Savings.
Conservation laws with discontinuous flux functions or systems of conservation laws are of great interest, since they arise in flows in heterogeneous media or flows driven by singular source terms, in the water flooding model in the petroleum industry, in modelling continuous sedimentation used in waste water treatment plants. Flow in non-heterogeneous anisotropic media can be modelled as stochastic PDE. The corresponding actions within Environment and Climate theme are Waste management and Water Resource Management. Microlocal analysis plays an important role in models coming from Earth Science while time frequency analysis is essential for reconstruction of climate and environmental dynamics. All mentioned models are interesting both from analytical and numerical point of view.



SWOT Analysis

The principal **STRENGTH** of the Center for Mathematical Research of Nonlinear Phenomena is concentration of well motivated and dedicated researchers with good results in their research areas. This is recognized on the national level and confirmed by the state by the fact that the Center is a **center of excellence** (one of two officially recognized in Serbia at this moment, the other one being in physics) according to the decision of the Ministry and the National Council for Science. Three research teams working within Center are well defined and yet close enough that synergy of efforts should yield added value in research and open new research possibilities.

A number of research topics developed CMRNP is a point of attraction for scattered research within Serbia. All research teams developed international cooperation and topics are relevant for mathematics within ERA. The Center has **critical mass of good quality people with creativity and innovative thinking**. The number of permanent researchers provides enough potential for the Center to become innovative engine and provide sustainable up-scaling of mathematical research.

The **leaders of the Center and research teams** are entusiastic high quality researchers with enough management experience. Their active participation in several international organizations and national policy teams (like National Council for Science – Dr. Pilipović), expert teams (National Committee for Mathematics – Dr. Kurilić) imply certain level of influence on **scientific policy** in Serbia.

The efforts made towards developing cooperation on international level resulted on contacts within ERA for all research teams and thus provide a necessary reference point for quality

assurance and ability to **generate state-of-the art research results**. Further more members of the Center are active in several international mathematical organization like AMS, LMS, GAMM, ISAAC, SIAM and participate in working bodies in some of them like ECMI (European Consortium for Mathematics in Industry) – Dr. Krejić, Society for Generalized functions – Dr. Pilipović, ISAAC (The International Society for Analysis, its Applications and Computation) - Dr. Pilipović. The majority of team leaders are already conducting research with international partners although this cannot be claimed for all researchers in the Center.

The researchers of the Center already have **experience with several international projects**, among them the following projects were successfully implemented: Tempus CD JEP project Mathematics for Technological Development coordinated by Dr. Krejić, several research projects „Pavle Savić"financed by Serbian and French Ministries of Science - coordinated by Dr. Pilipović, Dr. Kurilić and Dr. Nedeljkov, Neigbourhood programme Serbia-Hungary, coordinated by Dr. Takači, several WUS Austria CD projects, DAAD Project in the framework of the Stability Pact for South East Europe Center of excellences for applications of mathematics, 2002-2010. UNESCO International Basic Science project. At this moment three projects are financed by MNTR and all researchers at the Center are actively participating in them. A number of **scientific events** was organized by the members of Center, among them 12th Serbian Congress of Mathematics in 2008, Generalized Functions in 2004, 14th General Meeting of EWM (European Women in Mathematics), the bi-annual national conferences on Applied Mathematics (from 1993-) and several smaller workshops and winter-summer schools for international participants.

The members of the Center act as dynamic actors ready to accept **new trends and ideas and implement** them in the region. One of such efforts that is currently underway is an effort towards applicable mathematical research. Although well developed on European scale and in other developed countries like USA or Asian countries, it is practicly nonexisting in our region. Ability to recognize new developments and put an effort of implementing good EU practice in our region is a strong point of CMRNP.

The number of **young people**, PhD students as well as young researchers provide a lot of potential that should be unlocked within this project and yield a transformation of the Center from regional institution into an important point at ERA mathematical and scientific map. The total number of PhD grants for mathematics in Serbia is 11 at this moment and 7 of these grant holders are preparing their thesis at the Center.

The Faculty of Sciences has awareness of the importance of collaboration with SNE sector which is implied also in CMRNP, as its functional entity. There is high **awareness** of Triple Helix model and the importance of the building the system in which it can be successfully implemented and realized. Center has all the elements of such collaboration (recognized by the goverment, encouraging collaboration with industry, and atthe sam etime rerpresenting academia). UNSPMF (its Department of Mathematics and Informatics) is a member of the Serbian Sofware cluster (SMEs, institutes and faculties).

However there is a number of **WEAKNESSES** and they will be addressed within this project. First of all the Center is relying heavily on **financial support from the Ministry of Science** and is not financially independent. At this moment the Center is being financed through 3 scientific projects and some additional (insuffiecient) support from the Ministry. That is clearly not enough to transform the Center into ERA important point and diversification of sources of research funds is crucial. This should be achieved by increased capacity of researchers at the Center to gain scientific and industrial projects.

Second weakness is typical for Serbian institutions although to smaller extent at CMRNP than in average Serbian research institution. The scientists seriously **underestimate the importance of networking**. That is particularly emphasized within younger permanent staff and PhD students. Further more the attitude towards attending conferences and other scientific events is not very productive and should be changed. As a consequence scientific events are

not used to establish contacts with other researchers and our researchers are inclined to underestimate or overestimate their own results and quality. The existing international cooperation, although well developed with some of the most distinguished people in the corresponding research areas is mainly of „one-to-one“ character and does not have an institutional framework. In other words the main people are networked but CMRNP is not. As a consequence there is no smooth institutional inflow-outflow channel for exchange of knowledge which is necessary for brain gain, new ideas and established position of the Center in ERA. Overcoming this weakness will be an important tasks of this project.

Although initial efforts are already made the **cooperation with other stake-holders** (industry, SMEs,public companies) **is far from satisfactory**. Through cooperation with ECMI we already gained some knowledge in this area and made initial steps but that is clearly not enough.

Finally the **existing research equipment is not powerful enough** for new research directions planned by a number of research teams relevant to the applicable models. The Center (within DMI and UNSPMF) is able to provide only incremental upgrade of computational resources. Therefore the acquisition of equipment for the new computational laboratory planned in this project would provide a chance for an important step forward in the research capacity of the Center.

A number of **OPPORTUNITIES** for unlocking the research potential and reinforcing excellence at CMRNP is present at this moment. First of all one can easily notice that fundamental mathematical research performed at the Center is **relevant in a number of FP7 themes** as well as in the proposed **National Strategy** of Scientific and Technological Development of Serbia. Therefore it is essential to strengthen efforts already intiated in applicable research and gain necessary know-how for successful project preparation within FP7. Further more the Center has a chance to take the leading role in applied high-quality mathematical research within thematic priorties according to the National Stategy. Development of centers of excellence is listed in National Startegy as a preferred policy. The Ministry is providing lot of **support for scientists to join ERA** through training events and advice. Further more UNSPMF recently established an office for international cooperation and thus some administrative burden is taken away from researchers.

Through training and networking research skills will be improved resulting in larger scientific production but also in skills necessary for attracting other stake holders like industry and SMEs. Several **large** international **companies** are now present in Serbia as well as a number of medium size European companies and banks. Foreign companies brought an increase in awareness that research is utility for development. This awareness should be used to promote **applicable part of mathematical research**. With some of these stake holders we already have some kind of contact or cooperation – either providing training or applying some mathematical knowledge to their problems. But establishing more advanced, permanent and ongoing **cooperation** would be of great importance given that advanced mathematical models play an important role in their production and distribution services and several research themes developed in CMRNP is the key ingredient of such models (PDEs, optimization, stochastic PDEs, optimization under uncertainty, fractional calculus).

Brain gain is a recognized priority at the state level, in particular directed towards former nationals. Organization of international events will also provide opportunity for increased visibility. The Ministry of Science is providing **grants for PhD students** which allows them to concentrate on research fully and thus ensure **brain gain and sustainability** of efforts made at CMRNP at this moment.

The **THREATS** for CMRNP are mainly driven by the present socio-economic enviroment in Serbia which is essentially the same for the whole region. The level of general awareness of possible benefits that reserach can bring to the country as a whole is low and significant effort is necessary to increase the awarness of all stake holder. In particular the Triple Helix model consisting of three pillars: 1) state represented by goverment and its agencies; 2) scientifical

and academic community and 3) industry, is at the early stages its of application and advances very slowly. The partnership between these three pillars is yet to be established and the efforts made so far are mainly of declarative nature. Mathematics in particular is being seen as a purely fundamental science without any practical relevance and this kind of thinking is strongly present. Further more, there is a lack of knowledge of opportunities provided by international funding agencies and EU and hence poor awareness of opportunities within FP7. The general mind setting among researchers is not oriented towards searching research opportunities and funds for research. Time and resources necessary for preparation of project proposals and project management are not considered as high priority and hence not enough effort is directed towards success in this area. The situation is the same is true when it comes to establishing collaboration with local stake holders. Global crisis has affected Serbia more than expected and there is a lack of funds in all areas. As known, science and technological development are first on the list when it comes to governmental investments in the period of crisis. Therefore, the EU funds can be seen as the opportunity and encouragement in Serbian efforts to join EU.

CMRNP has the ability to **successfully overcome** the above mentioned threats. First of all we already posess experience in applying for international project and implementing them as well as in organizing the international events. These abilities will be further developed within MATH POST. We are aware of the importance of Triple Helix model for successful development and have already taken some initial steps towards establishing the triangle partnership. But these steps are clearly not enough and one of the most important takst at this project will be to gain knowledge and experience from EU networking partners in this area. Through the available EU funds we plan to conduct the initial activities and create the fundamental base for further development of skills, abilities and equipment. Center will hopefully become the part of the EU networked systems and will be able to conduct its activities together within network partnership.

Project Objectives

The objective of this project is to provide significant incentive to fully develop the research potential of CMRNP, to increase its visibility within ERA, to promote excellence and applicability of mathematical research, to ensure brain gain, to increase the research opportunity and capacity of research for end users through acquiring new equipment and to ensure sustainability of the results achieved in this project.

Action Plan

A set of five coherent measures will be implemented within this project to ensure significant decrease of weaknesses and possible treats as well as full employment of strong points of the Center into taking the existing opportunities for further development. The measures to be implemented are the following.

- **Exchange of know-how and experience**

A number of secondments, short-time training visits, and fact-finding two-way visits are planned within the project. The choice of EU partners provide an excellent pool of knowledge and experience that will be fully used The partners are Oxford Mathematical Institute (OMI), Department of Mathematics at Humboldt University (DMH), Faculty of Mathematics at University of Vienna (FMV), CASA (Center for Analysis, Scientific computing and Applications) at Eindhoven University of Technology and Department of Mathematics at University of Torino (DMT). All these partners are outstanding research units able to provide experience and know-how; there is a significant overlap in research interest with the Center and contacts that will be transformed into permanent networking. All partners have vast experience in applicable research and management of collaboration with other stake holders like industry and SMEs and hence will be able to provide transfer of knowledge to CMRNP.

- **Recruitment of experienced researchers**

The present research themes are well covered by the existing staff at the Center. Furthermore the number of young people (PhD students) doing research at the Center provides substantial

pool of research potential for development. Hence we do not plan to hire experienced researchers within this project. However brain gain obtained by fresh ideas and experienced people is an imperative for development in science. Therefore we propose establishing of **Visiting Seminar** within this project. The Seminar will provide opportunity to host a number of experienced researchers, coming from different research areas relevant to the Center and thus give incentive to the development of all groups. The visitors will be staying at the Center for nontrivial time, conducting research and sharing their knowledge and experience with the researchers at the Center. Particular attention will be devoted to former nationals successfully established at the international level so the seminar will have **pull-back** component. Initial screening among former nationals and other contacts assures us that such seminar will generate lot of high quality visitors and thus provide significant brain gain, more beneficial than hiring a modest number of experienced researchers during the project lifetime. This is an action which will be ongoing and traditional and therefore will provide sustainability of the action.

- **Acquiring of new research equipment**

New computational laboratory will be acquired. This upgrade of research capacity will be crucial for several research directions. Already mentioned targets in applicable research that will be the focus of this project include conservation laws, fluid dynamics, stochastic optimization, semismooth problems of large dimensions, PDE constrained problems and stochastic PDEs. For all these problems massive computation and simulation is necessary.

- **Organization of workshops and conferences**

Five workshops are planned within the project. The form will be as usual in mathematics – combination of training event and scientific conference. The well defined topics will serve as a training event where the world top researchers will provide training in the latest development. Participant will at the same time have a chance of presenting their own results and establish closer contacts, discuss the results and exchange new ideas. These workshops will be international, heavily advertised through professional networks and thus will also increase the visibility of the Center. One workshop will be devoted to Mathematical Models in Real-Life targeting other stake holders – industry, SMEs, public companies as well as researchers with the aim of increasing awareness of socio-economic benefit of applicable mathematical research and identifying research possibilities that will yield successful collaboration. Participation of the researchers at the Center at **important international conferences** will be an important part of the strategy for network building and exposing them to international environment as well as increasing visibility of the Center.

- **Dissemination and promotional activities**

To ensure visibility of the Center's excellence and know-how, its activities within EU and elsewhere as well as to promote the outcome of this project a set of dissemination and promotional activities will be placed. Four target groups are distinguished and a set of measures is planned for each one of them. The projects results will be heavily advertised to policy makers using the active role some of the Center's research play in policy bodies as well as the advice provided by the Steering Committee.

1.2 Quality and effectiveness of the support mechanisms and associated work plan

Overall strategy of the work plan within this project consists of six different work packages designed to ensure **successful development of the research potential** at CMRNP and to provide its transformation from national and regional leader into a significant point at ERA map. The work packages are planned in details and discussed with EU partners to ensure its smooth realization and maximize the benefit for CMRNP as well as to provide benefit to EU partner institution through established research cooperation. Each work package has the responsible person chosen among the researchers at the Center taking into account already existing

experience and personal skills. The vast number of planned activities requires mobilization of all researchers at the Center and should provide beneficial increase in enthusiasm and additional incentive for all research teams. All research teams in the Center are involved in the project, the choice of partners is based on their excellence and intersection of research interests with the Center so synergy results are to be expected.

Personal and professional development of the researchers at CMRNP will be subject of several actions – networking, training visits, participation at international conferences and workshops organized at the Center. Increased visibility within ERA will be achieved through high profile plenary lecturers at the workshops and international participants. Furthermore presentations of CMRNP results at international conferences will serve the same purpose.

Acquisition new equipment will increase research opportunities at the Center allowing a sequence of new research directions that require heavy computation and simulation effort. At the same time broadening of research scope as well as collaboration with EU partners will stimulate a slight shift towards more applicable mathematical research. Thus more opportunities for the Center within FP7 will open, Center will interact more actively with other stake holders and the contribution to the socio-economic needs of the country will be improved.

Vast dissemination plan is developed targeting all relevant groups. The final results will be increased awareness of general public of the importance of research, established links with other stake holders, and increased visibility of CMRNP in the international mathematical community and influence on scientific policy in Serbia.

In order to ensure smooth realization of the project, management will be done with utmost care. The project coordinator together with the Management Team will take care of the project on daily base. Each EU partner has a contact person that will provide necessary support in organizational matters concerning training, networking and organization of workshops.

Steering Committee will play an important role in providing quality control, monitoring of the Action Plan, helping to build more effective links with other stage holders, national scientific policy influence as well as providing expertise to the research agenda beyond the project's end.